

Activating Feedback in Formative Assessment : From Receptive to Active Learning with Automated Feedback



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overview

1. examples
2. theory
3. suggestions



examples

Give a quadratic expression
which has exactly the two roots -3 and -1 .

$$f(x) = (x-3)(x-1)$$

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer...

examples

$$\frac{1}{4} = \frac{5}{20}$$

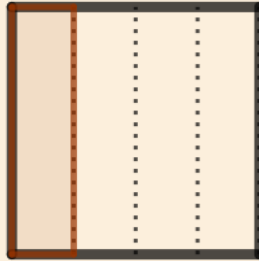
because:

$\frac{1}{4}$ has been expanded

by the number .

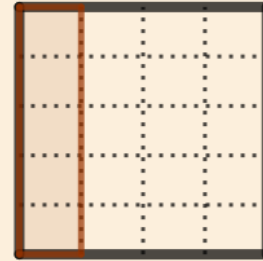
Correct!

Because, as you can see,



$$\frac{1}{4}$$

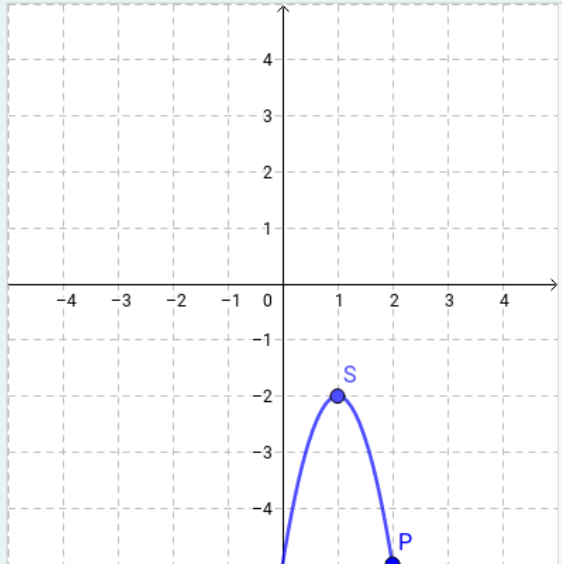
$$\frac{.5}{.5}$$



$$\frac{5}{20}$$

examples

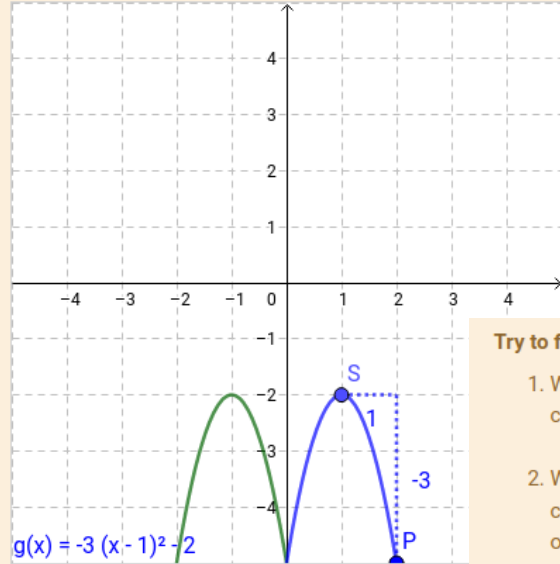
Move the points S and P,
such that the graph fits with
 $f(x) = -3 \cdot (x + 1)^2 - 2$.



Check

Wrong, too bad!

The green graph would be correct.



Why?

You can find out yourself.

Correct your blue graph
and watch how the expression changes.

Try to find answers to the following questions:

1. Where in the expression
can you see the coordinates of the vertex?
2. Where in the expression
can you see a value for the opening
of the parabola?

Do you have an idea already?

Then try the task again.

Or wait 30 seconds
after which a full solution will appear:

Musterlösung

overview

1. examples
2. theory
3. suggestions



a short digression
into
AuthOMath



AuthOMath

AuTo

- a moodle based authoring tool for randomized interactive and dynamic multimodal mathematical tasks with automatic adaptive feedback

DiCo

- a didactical concept for designing online based interactive learning material for use in mathematics teacher education



AuthOMath

This visualization shows how two fractions are added.

Translate into maths:

$\square + \square = \square + \square$

$= \square + \square = \square + \square$

$= \square = \square$

```
n1:rand([2,3,4,5]);  
n2:rand_with_prohib(2,5,[n1]);  
z1:rand(n1-1)+1;  
z2:rand(n2-1)+1;
```

```
1 <br>This visualization shows<br>how two fractions are added.<br><br>Tra  
2 |  
3 <table style="border-collapse: collapse; width: 400px;" border="0">  
4   <tbody>  
5     <tr>  
6       <td style="width: 180px; vertical-align: bottom; border-sty  
7  
8         [[geogebra set="n1,n2,z1,z2,x1,y1,x2,y2"]]  
9         params["material_id"] = "yqhjpr2c";  
10        params["width"] = 450;  
11        params["height"] = 550;  
12        params["borderColor"] = "rgba(0, 0, 0, 0)";  
13        params["transparentGraphics"]= true;  
14        params["scale"] = 0.5;  
15        [[/geogebra]]  
16  
17        </td>
```


AuthOMath

names of variables in applet, with

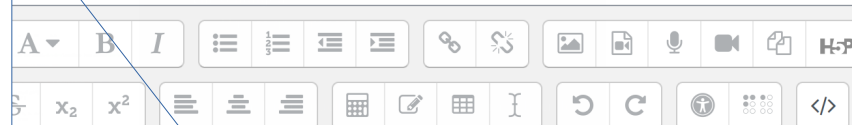
set: transmit values from STACK to applet
watch: read values from applet into STACK on "Check"
remember: remember values for reloading applet

applet ID on geogebra.org

GeoGebra App Parameters

https://wiki.geogebra.org/en/Reference:GeoGebra_App_Parameters

```
n1:rand([2,3,4,5]);  
n2:rand_with_prohib(2,5,[n1]);  
z1:rand(n1-1)+1;  
z2:rand(n2-1)+1;
```



>This visualization shows
how two fractions are added.

Tra

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      params["scale"] = 0.5;  
      [[/geogebra]]  
17 | </td>
```

overview

1. examples
2. theory
3. suggestions

for more on
AuthOMath,
cf. www.authomath.org



overview

1. examples
2. theory
3. suggestions



feedback

...is information
about performance

...its function is
assisting learning

...hence should
be perceived as
advice for action

parameters

width of focus

...its function is
assisting learning

...hence should
be perceived as
advice for action

parameters

width of focus

grade of adaption

grade of activation

...hence should
be perceived as
advice for action

parameters

width of focus

grade of adaption

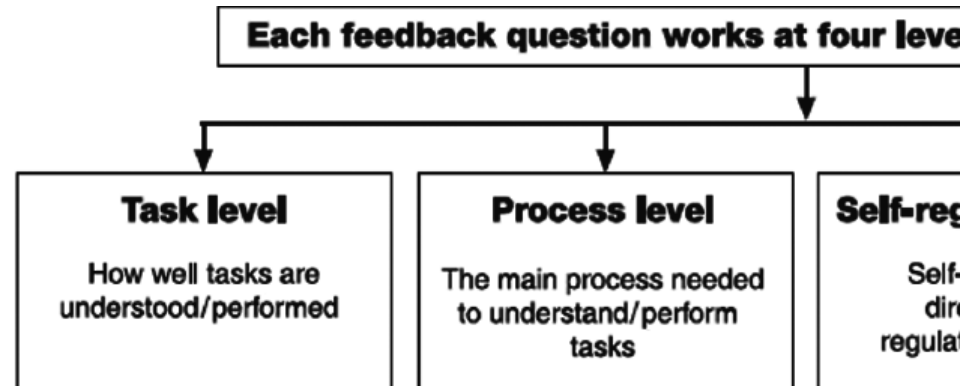
grade of activation

focus : idea

- from procedures to underlying concepts
- from addressing procedures that are necessary to master the given task to providing the conceptual basis for understanding the given and related tasks

focus : idea

- from procedures to underlying concepts



focus

- from procedures to underlying concepts

Calculate:

$$\frac{1}{2} + \frac{1}{5} = \boxed{2/10}$$

Wrong, sorry!

You have found a common denominator.
But also expand the numerators:

$$\frac{1}{2} + \frac{1}{5} = \frac{1 \cdot 5}{2 \cdot 5} + \frac{1 \cdot 2}{5 \cdot 2}$$

focus

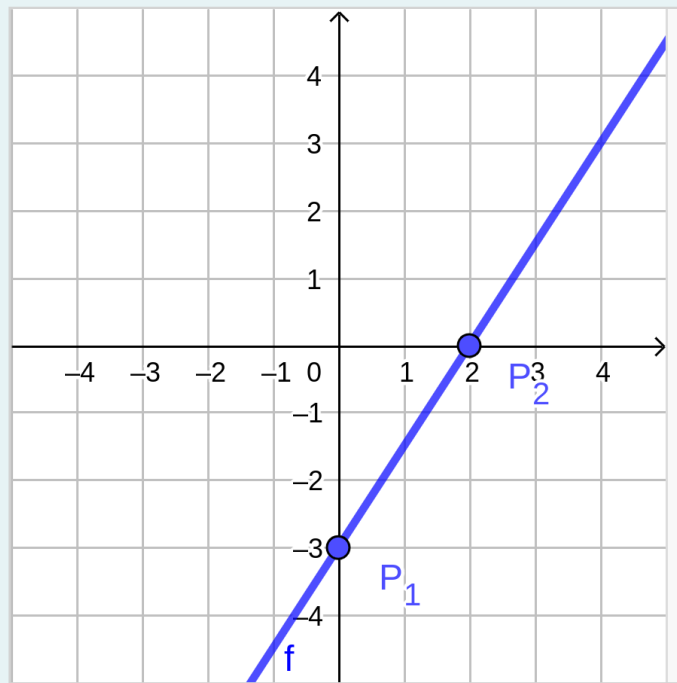
- from procedures to underlying concepts

Give the graph to the function

$$f(x) = 2 \cdot x - 3.$$

Place P_1 and P_2

such that the line fits the expression.



Follow these steps:

1. Place P_1

The number -3 in $f(x) = 2 \cdot x - 3$ marks the place on the y -axis.

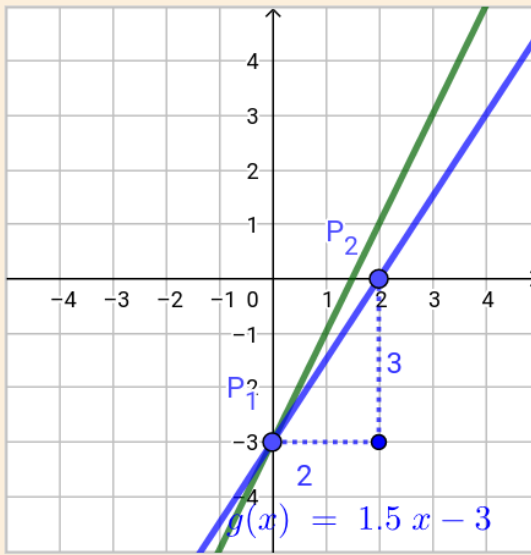
Place P_1 here.

2. Place P_2

The other number 2 in $2 \cdot x - 3$ denotes the slope of the line.

Hence start with P_2 in P_1 , then move P_2 one step to the right, and after that move 2 steps vertically.

Place P_2 here.



focus

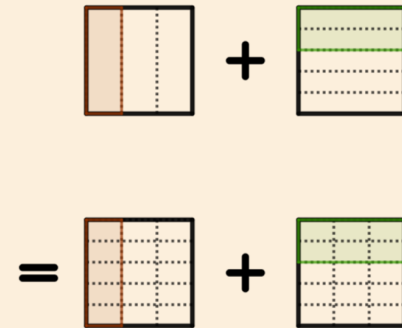
- from procedures to underlying concepts

Calculate:

$$\frac{1}{3} + \frac{2}{5} = \boxed{3/8}$$

Wrong, sorry!

Maybe this visualisation of the first step helps you to find your mistake?



focus

- from procedures to underlying concepts

Solve:

$$2 \cdot (q + 1) = 4$$

Copy the equation below, then note each next step beneath:

$$2 \cdot (q + 1) = 4$$

$$2 \cdot q + 2 = 4$$

$$2q = 2$$

$$q = 1$$

$$L = \{ 1 \}$$

Good. Your solution is correct.
And the transformations are fine.

But that took long!

There is a faster solution - compare:

$$2 \cdot (q + 1) = 4$$

$$2 \cdot q + 2 = 4$$

...

$$2 \cdot (q + 1) = 4$$

$$q + 1 = 2$$

...

One is your strategy, the other is faster.

Well? Do you have an idea?

Then try this task again.

Else wait for 30 sec, then a full solution appears!

[Click here for full solution](#)

Here you have two full solutions:

Both are correct.

$$2 \cdot (q + 1) = 4$$

$$2 \cdot q + 2 = 4$$

$$2 \cdot q = 2$$

$$q = 1$$

$$2 \cdot (q + 1) = 4$$

$$q + 1 = 2$$

$$q = 1$$

Choose the more efficient and **try again!**

[Try another question like this one](#)

focus : feedback content

- from procedures to underlying concepts
- worked out solving procedure
- specific reference to single steps
- interactive scaffolding through steps
- references to relevant rules
- explanatory models (“Grundvorstellungen”)
- representational or contextual flexibility (e.g. geometric visualisations, numeric examples, familiar contexts from outside maths, if not part of the task)
- strategic flexibility

focus : think about it

- from procedures to underlying concepts

*“a deep understanding of learning involves the construction of meaning (understanding) and relates more to the relationships, cognitive processes, and transference to other more difficult or untried tasks”
(Hattie & Timperley, 2007)*

mastery of procedures reduce cognitive load while solving complex and challenging problems

parameters

width of focus

grade of adaption

grade of activation

adaption : idea

- from nearly none to very differentiating
- the same feedback regardless what the (wrong) answer is
- specific feedback for each answer case

adaption

- from nearly none to very differentiating

Give a cubic expression which has exactly the two roots 1 and 4 .

$$f(x) = (x-4)*(x-1)$$

Wrong, too bad.

A correct expression would be $(x - 4)^2 \cdot (x - 1)$.

Why is that?

You need to know

that a linear expression like $(x - a)$ has a as root,

that $(x - a) \cdot (x - b)$ is a quadratic expression and has a and b as roots,

and that $(x - a) \cdot (x - b) \cdot (x - c)$ is a cubic expression with roots a, b and c .

adaption

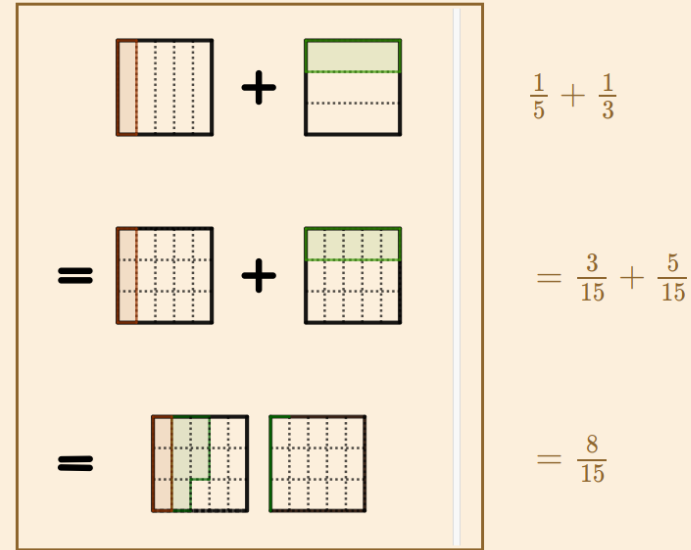
- from nearly none to very differentiating

Calculate:

$$\frac{1}{5} + \frac{1}{3} = \boxed{2/8}$$

Wrong, I am afraid.

This visualization should help you to understand:



And also reduce the fraction, if necessary.

adaption

- from nearly none to very differentiating

Calculate:

$$\frac{1}{2} + \frac{1}{5} = \boxed{2/10}$$

Wrong, sorry!

You have found a common denominator.

But also expand the numerators:

$$\frac{1}{2} + \frac{1}{5} = \frac{1 \cdot 5}{2 \cdot 5} + \frac{1 \cdot 2}{5 \cdot 2}$$

adaption

- from
nearly
none

to very
differentiating

Give a quadratic expression
which has exactly the two roots -3 und -1 .

$$f(x) = (x-3)*(x-1)$$

NEARLY correct, but not quite!

You seem to know what to do.

Just check your answer...

adaption : content

- from nearly none to very differentiating
- basic procedural and/or conceptual knowledge for mastering all varieties of the task
- specific advice (procedural or conceptual) for a priori identified answer cases:
 - correct,
different in strategies
 - wrong,
different as to systematic errors or misconceptions

adaption : think about it

- from nearly none to very differentiating

adaption supports acceptance and certainty about how to proceed

in retention tasks, specific feedback is superior to general advice.

in transfer tasks, no difference between specific and general advice

parameters

width of focus

grade of adaption

grade of activation

activation : idea

- from receptive to active
- from informing about (parts of) the necessary knowledge to prompting the learner to (re)construct the necessary knowledge by him/herself

activation

- from receptive to active

As you know,

$$(a + b)^2 = a^2 + 2 \cdot a \cdot b + b^2$$

$$(a - b)^2 = a^2 - 2 \cdot a \cdot b + b^2$$

$$(a - b) \cdot (a + b) = a^2 - b^2$$

Now factorise $18 \cdot s^2 + 24 \cdot s \cdot t + 8 \cdot t^2$ by using one of the three formulas above.

You can do your calculations here:

$$\begin{aligned} &18*s^2+24*s*t+8*t^2 \\ &= (18*s+8*t)^2 \end{aligned}$$

Denote your solution here:

$$(18*s+8*t)^2$$

Wrong, too bad.

Correct would be $2 \cdot (3 \cdot s + 2 \cdot t)^2$

That's how to do it:

Here is the expression again:

$$18 \cdot s^2 + 24 \cdot s \cdot t + 8 \cdot t^2$$

First, you need to find two square numbers.

You can identify them once you factor out 2:

$$= 2 \cdot (9 \cdot s^2 + 12 \cdot s \cdot t + 4 \cdot t^2)$$

Now the square numbers are visible inside the brackets: 9 und 4

Second, choose from the three formulas mentioned above the one that has the same structure as the expression inside the brackets:

$$9 \cdot s^2 + 12 \cdot s \cdot t + 4 \cdot t^2$$

corresponds to

$$a^2 + 2 \cdot a \cdot b + b^2$$

Third, identify the corresponding parts of each expression:

a^2 corresponds to $9 \cdot s^2$, hence $a = 3 \cdot s$, and

b^2 corresponds to $4 \cdot t^2$. So $b = 2 \cdot t$

And check whether $2 \cdot a \cdot b$ corresponds to $12 \cdot s \cdot t$:

$$2 \cdot 3 \cdot s \cdot 2 \cdot t = 12 \cdot s \cdot t,$$

which hence is the case.

Fourth, substitute the values for a and b in $(a + b)^2$.

And do not forget the factor from the first step to denote the final solution:

$$= 2 \cdot (3 \cdot s + 2 \cdot t)^2$$

Try this task again!!

activation

- from receptive to active

Write $\frac{3}{4}$ as a decimal number.

$$\frac{3}{4} = \boxed{3.4}$$

Tip

Follow these steps

1. Expand the fraction so that the denominator is 10 or 100 or 1000...
2. Count the number of zeros in the new denominator.
3. Formulate the decimal number.

Do you know now what to do?

Change your solution above and click on "check".

Else wait 30 sec for "more help" below.

[more help](#)

Fill the blanks:

1. Expand the fraction such that the denominator is 10 or 100 or 1000...

Expand $\frac{3}{4}$ by

$$= \frac{3 \cdot 25}{4 \cdot 25}$$

= (enter a fraction here)

Count the number of zeros of the new denominator.

The denominator of $\frac{75}{100}$ has zero(s). (enter a number here)

3. Formulate the decimal number

$\frac{75}{100}$ in the form of a decimal number:

activation

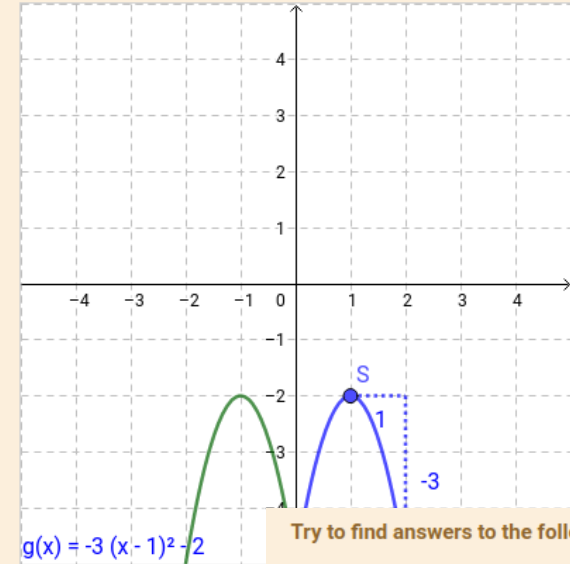
- from receptive to active

Move the points S and P, such that the graph fits with $f(x) = -3 \cdot (x + 1)^2 - 2$.

Check

Wrong, too bad!

The green graph would be correct.



Try to find answers to the following questions:

1. Where in the expression can you see the coordinates of the vertex?
2. Where in the expression can you see a value for the opening of the parabola?

Do you have an idea already?

Then try the task again.

Or wait 30 seconds after which a full solution will appear:

Musterlösung

activation

- from receptive to active

Give a quadratic expression which has exactly the two roots -3 and -1 .

$$f(x) = (x-3)*(x-1)$$

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer...

activation : content

- from receptive to active
 - statements, propositions, description
 - pictures, graphs
 - videos, movies
- clozes, scaffolding
 - questions, hints, food for thought
 - interactive elements for exploration

activation : think about it

- from
receptive

to
active

“Interactive feedback is more effective than other kinds of feedback in improving students’ performance.”

“Unless students see themselves as agents of their own change, and develop an identity as a productive learner who can drive their own learning, they may neither be receptive to useful information about their work, nor be able to use it.”

for experts,
corrective or thought provoking feedback
seems sufficient

for novices,
scaffolding or worked out examples
are needed

parameters

width of focus

grade of adaption

grade of activation

...and structure

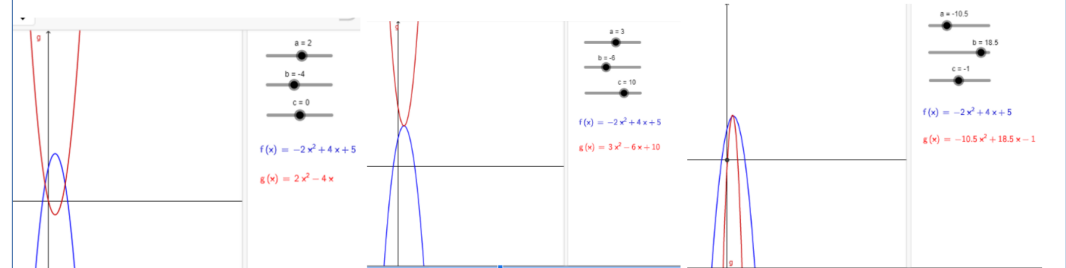
structure

location
order
timing

- as part of task

Create three example sets of functions f and g that follow as many of the following conditions as possible:

- (1) the graph of $f(x)$ intersects the graph of $g(x)$ in exactly one point;
- (2) the two functions have the same symmetry axes;
- (3) $g(x)$ passes through the origin $(0,0)$ of the system; and
- (4) the function $g(x)$ has a minimum.



structure

location
order
timing

- as part
of task

Give an example of a set of vectors that span \mathbb{R}^3 Tidy STACK

✓ **Correct answer, well done.**
This set spans \mathbb{R}^3

Give another example of a set of vectors that span \mathbb{R}^3 , that does not contain the standard basis vectors.

✓ **Correct answer, well done.**
This set spans \mathbb{R}^3

Give an example of a set of more than 3 vectors that span \mathbb{R}^3 . If no such example exists enter none.

structure

location
order
timing

- as part of task
- immediately after task

Give a quadratic expression
which has exactly the two roots -3 and -1 .

$$f(x) = (x-3)*(x-1)$$

NEARLY correct, but not quite!

You seem to know what to do.

Just check your answer...

structure

location
order
timing

- as part of task
- immediately after task
- delayed (in bits)

Give a quadratic expression which has exactly the two roots -3 and -1 .

$$f(x) = (x-3)(x-1)$$

NEARLY correct, but not quite!

You seem to know what to do.
Just check your answer...

Here is how:

You need to know:

An expression like $(x - a) \cdot (x - b)$ is quadratic and has a and b as roots.

To have -3 and -1 as roots
 $(x + 1) \cdot (x + 3)$ would fit.

Try again!

structure

location
order
timing

- as part of task
- immediately after task
- delayed (in bits)

The lines g and h are parallel.

Think of point D movable on h .

With D , the p
such that AE

h

g

Now think of
such that ang

What then is t

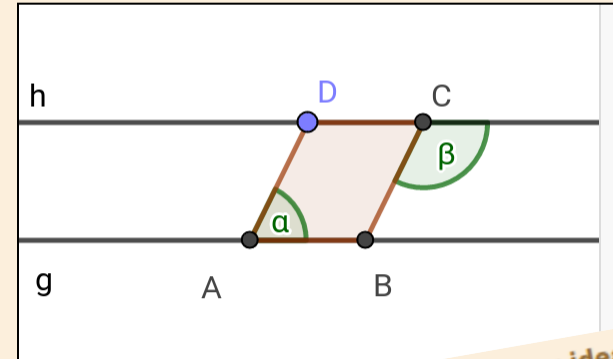
$\beta =$

Wrong, sorry.

Correct is $\beta = 150^\circ$.

Why?

Find out yourself,
by moving D in real now:



Compare α and β .

How do these two relate?

Do you have an idea already?

Then try the task again.

Or wait 30 seconds

structure

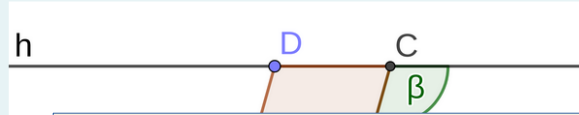
location
order
timing

- as part of task
- immediately after task
- delayed (in bits)

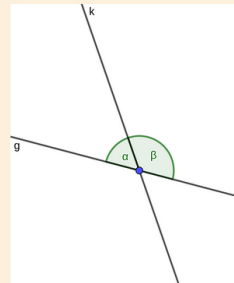
The lines g and h are parallel.

Think of point D movable on h .

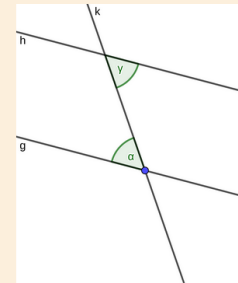
With D , the point C moves on h such that $ABCD$ stays a parallelogram.



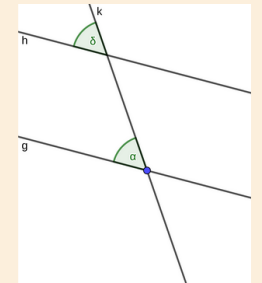
This you need to know:



α and β are adjacent angles. Hence they add up to 180° .



α and γ are alternate angles. Hence they are of equal size.



α and δ are step angles at the parallels g and h . Hence they are of equal size.

Do you have an idea already?
Then try the task again.
Or wait 30 seconds

structure

location
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timing

- as part of task
- immediately after task
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The lines g and h are parallel.

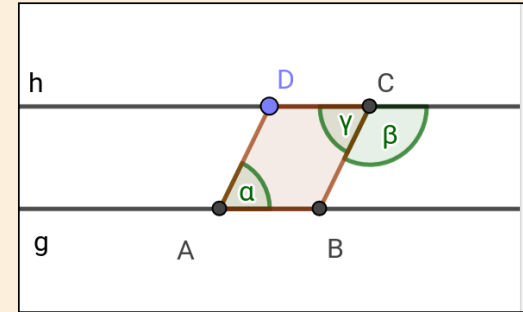
Think of point D movable on h .

With D , the point C moves on h such that $ABCD$ stays a parallelogram.

h

Again you can move D here.

g



Now this
such that

What then

$\beta = 30$

In this figure γ was added.

What can you say about γ and β now?

Which of the three statements from the second hint applies here?

Use the other two statements too
to find a relation between α and β .

Do you have an idea already?
Then try the task again.
Or wait 30 seconds

structure

location
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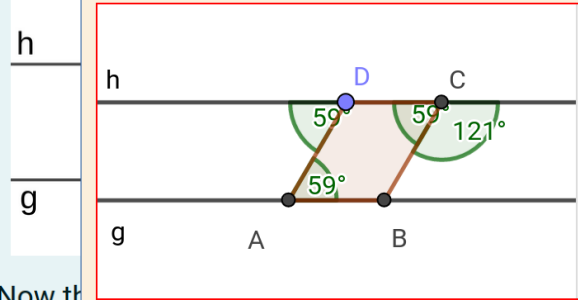
- as part of task
- immediately after task
- delayed (in bits)

The lines g and h are parallel.

Think of point D movable on h .

With D Move the point D now.

such th



Now th

such th

What t

$\beta = 3$

And you can see that

- 1: angles at A and D are always of equal size,
- 2: both angles at D and the inner angle at C are always at equal size,
- 3: both angles at C add up to 180° .

Why is that?

- 1 is correct because both angles are alternating angles at the parallels g and h ,
- 2 is correct since both angles are step angles at the parallels g and h , and
- 3 is correct because both angles are adjacent angles.

Hence both angles at A and the exterior angle at C add up to 180° .

In short:

$$\beta = 180^\circ - \alpha = 180^\circ - 30^\circ = 150^\circ$$

structure

location
order
timing

*“Give a moment
to think it over...”*

*for low achievers, prompt timing,
for high achievers, delayed timing
of feedback seems suitable*

*when testing declarative knowledge
feedback only after second try
is more effective*

overview

1. examples
2. theory
3. suggestions



models

worked solution

Sorry, wrong
(KR)

Correct would be...
(KCR)

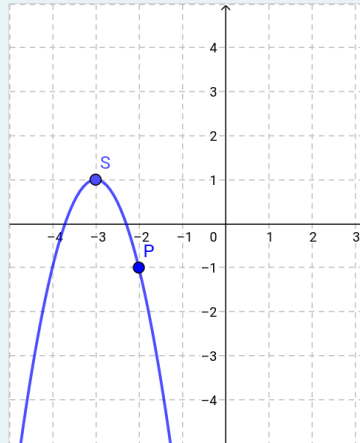
This is
how to do it
correctly:

...
(KH)

Try again?
Click here:

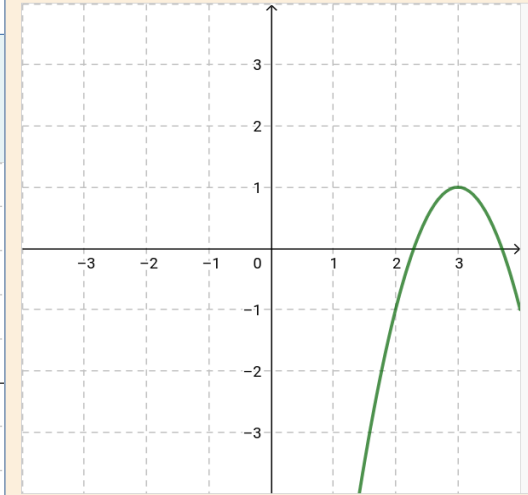
appears
without delay

Verändere die Position der Punkte S und P so,
dass der Graph zur Funktion f
mit $f(x) = -2 \cdot (x - 3)^2 + 1$ passt.



Leider falsch.

Richtig wäre der grüne Graph.



So geht's:

Die Funktionsgleichung lautet ja
 $f(x) = -2 \cdot (x - 3)^2 + 1$.

1. Platziere zuerst den Punkt S :

3 und 1 sind die Koordinaten des Scheitelpunktes.

Man findet sie im Term mit umgekehrten Vorzeichen in der Klammer
und als zuletzt angegebene Zahl.

Platziere also S so,
dass er die Koordinaten 3 und 1 hat.

2. Platziere jetzt P :

-2 steht für die Parabelöffnung.

Hierzu geht man von S einen Schritt nach rechts oder links
und dann 2 Schritt(e) nach unten.

models

error information

Sorry, wrong
(KR)

appears
without delay

You probably
made this error:

⋮
(KM)

or

The first correct
step would be

⋮
(KTC)

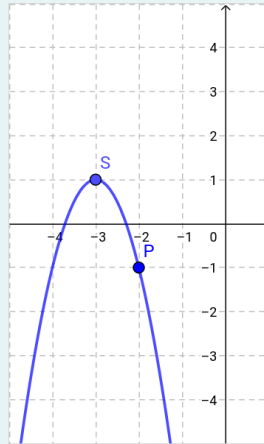
Try again?
Click here,
or wait 30 seconds
for a full solution:

appears
with 30" delay

This is
how to do it
correctly:

⋮
(KH)

Verändere die Position der Punkte S und P so,
dass der Graph zur Funktion f
mit $f(x) = -2 \cdot (x - 3)^2 + 1$ passt.



Leider falsch.

Du hast vermutlich
die Vorzeichen in $f(x) = -2 \cdot (x - 3)^2 + 1$
nicht beachtet.

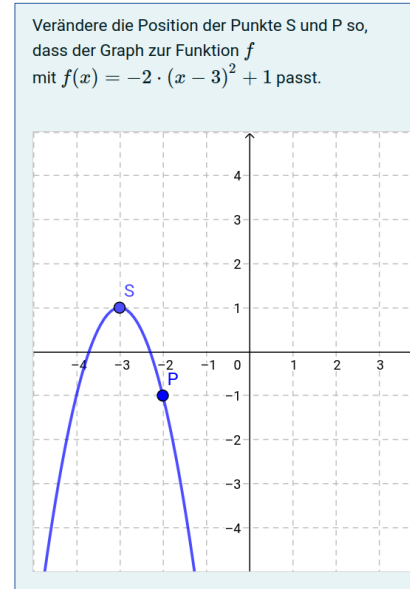
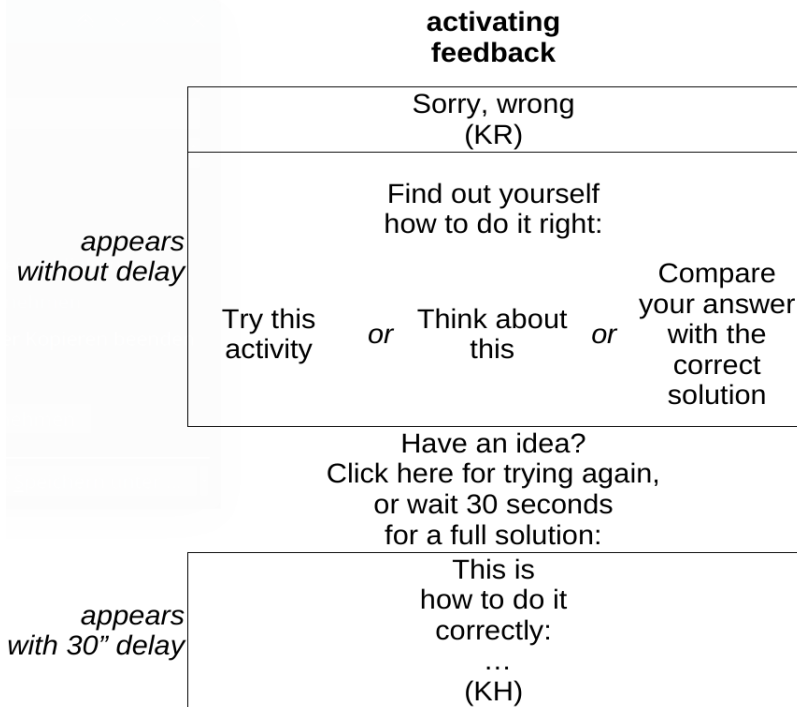
Und? Hast du schon eine Idee?

Dann versuche die Aufgabe noch einmal.

**Ansonsten warte 30 Sekunden,
dann erscheint hier eine Musterlösung:**

Musterlösung

models



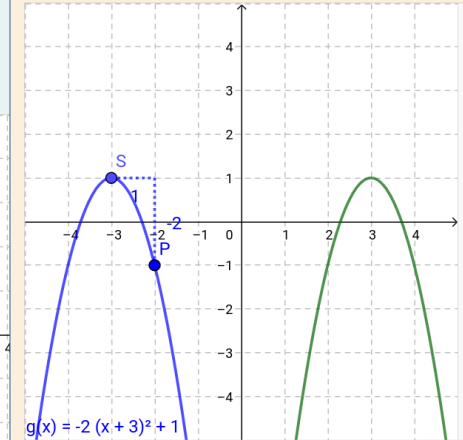
Leider falsch.

Richtig wäre der grüne Graph.

Warum?

Das kannst du selbst herausfinden

Korrigiere deinen blauen Graphen und achte darauf, wie der Term sich ändert!



Beantworte dabei für dich die folgenden Fragen:

1. Wo im Term erkennt man die Koordinaten des Scheitelpunkts?
2. Wo im Term erkennt man die Öffnung der Parabel wieder?
Die Öffnung ist übrigens die Länge der senkrechten Seite des gestrichelten Dreiecks, wenn die horizontale 1 lang ist.

Und? Hast du schon eine Idee?

Dann versuche die Aufgabe noch einmal.

Ansonsten warte 30 Sekunden, dann erscheint hier eine Musterlösung:

Musterlösung

summary

1. examples
2. theory
3. suggestions

- width of feedback focus
on procedural or conceptual knowledge
- grade of adaption
to student correct or wrong answers
- grade of activation
to foster change from receptive to active attitude
- structure and timing
to model sensible learning paths
- model 1: full solution
- model 2: error information
- model 3: activating feedback
- model 4: reference to explanatory models

